

Image Preprocessing 2

KKY/USVP Lecture 3

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EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání



DEPARTMENT OF
CYBERNETICS



Frequency Analysis

Fourier series

- ▶ for periodic signals
- ▶ the periodic signal $y(t)$ with period T can be expressed as the sum of sines and cosines of frequencies that are a multiple of fundamental frequency $f = 1/T$
- ▶

$$y(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} \left[A_n \cos \left(n \frac{2\pi}{T} t \right) + B_n \sin \left(n \frac{2\pi}{T} t \right) \right] \quad (1)$$

Frequency Analysis

Fourier transform

- ▶ it always exists - it is a generalization of Fourier series to an infinite interval
- ▶ Fourier transform for two variables (2D)
- ▶

$$F(u, v) = \iint_{-\infty}^{\infty} f(x, y) \cdot e^{(-2\pi i \cdot (xu + yv))} dx dy \quad (2)$$

- ▶ u, v ... spatial frequencies
- ▶ relation of Fourier transform and convolution
- ▶ the Fourier transform of a convolution is a product and the product is a convolution

Frequency Analysis

Discrete Fourier Transform (DFT):

- ▶ Used to calculate the Fourier transform of a sampled (discrete) function in discrete frequency points
- ▶ Discrete Fourier transform for two variables (2D)
- ▶

$$F(u, v) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} f(n, m) \cdot e^{-2\pi i \cdot \left(\frac{nu}{N} + \frac{mv}{M}\right)} dx dy \quad (3)$$

- ▶ DFT is very computationally expensive
- ▶ Fast Fourier Transform (FFT): Fast algorithm for calculating the Fourier transform

Mathematical morphology

Introduction

- ▶ separate area of image analysis
- ▶ based on point set theory
- ▶ in the center of attention is the shape of the objects
 - ▶ shape identification
 - ▶ optimal reconstruction of the shape that is broken
- ▶ easy hardware implementation, faster than the classic approach
- ▶ Images are usually first preprocessed using standard techniques, and objects are found by segmentation methods → binary image

Mathematical morphology

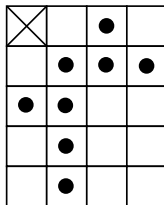
Application

- ▶ preprocessing (noise removal, simplification of object shape)
- ▶ emphasis on the structure of objects (skeleton, thinning, amplification, calculation of convex envelope, marking of objects)
- ▶ description of objects by numerical characteristics (area, perimeter, projection, etc.)

Mathematical morphology

Binary image = point set

- ▶ X - objects
- ▶ X_c - background (including holes in objects)



$$X = \{ (0, 2), (1, 1), (1, 2), (1, 3), (2, 0), (2, 1), (3, 1), (4, 1) \}$$

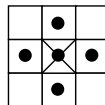
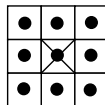


origin (0,0)

Mathematical morphology

structuring element

- ▶ relation with another, smaller point set B , called structuring ELEMENT
- ▶ frequently used structuring elements



- ▶ the origin does not have to be a point of the structuring element (see right element in the figure above)
- ▶ Elements that have the same properties for different directions are called isotropic

Mathematical morphology

Properties

- We imagine the morphological transformation as if we were moving the structuring element B systematically throughout the image W . The **point of the image** that coincides with the **origin of the structuring coordinates element**, we call the **instantaneous point**. The result of the relationship between the image and the structuring element we write to the instantaneous point of the image.

- For each morphological transformation $\Phi(x)$ there is a dual transformation $\Phi^*(x)$

$$\Phi(x) = (\Phi^*(x^C))^C \quad (4)$$

- **Basic transformations:** translation, dilation, erosion, opening, closing

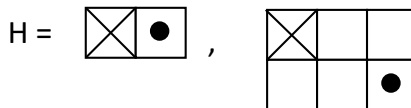
Mathematical morphology

Translation

- The translation of a point set X by a vector h is denoted by X_h

$$X_h = \{d \in E^2; d = x + h \text{ for } x \in X\} \quad (5)$$

- Example



Mathematical morphology

Dilation \oplus

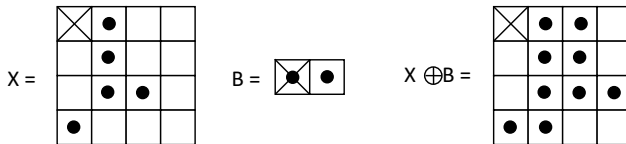
- merge two point sets using vector sum (Minkowski sum)

$$X \oplus B = \{d \in E^2; d = x + b \text{ for } x \in X, b \in B\} \quad (6)$$

- Example

$$X = \{(0, 1), (1, 1), (2, 1), (2, 2), (3, 0)\} \quad B = \{(0, 0), (0, 1)\} \quad (7)$$

$$X \oplus B = \{(0, 1), (0, 2), (1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 0), (3, 1)\} \quad (8)$$



Mathematical morphology

Dilation \oplus properties

- ▶ The most commonly used structuring element - 3x3, containing all 9 points of the eight-neighborhood
 - ▶ objects grow by one layer at the expense of the background
 - ▶ holes and bays with thickness of 2 points is filled
- ▶ Properties
 - ▶ commutative $X \oplus B = B \oplus X$
 - ▶ associative $(X \oplus B) \oplus D = X \oplus (B \oplus D)$
 - ▶ dilation can be expressed as the union of shifted point sets $X \oplus B = \bigcup_{b \in B} X_b$
 - ▶ translation is the dilation using structuring element that contains exactly one point
 - ▶ invariant with respect to displacement $X_h \oplus B = (X \oplus B)_h$

Mathematical morphology

Erosion \ominus

- merge two point sets using the vector subtraction
- is a dual transformation to dilation NOT INVERSE

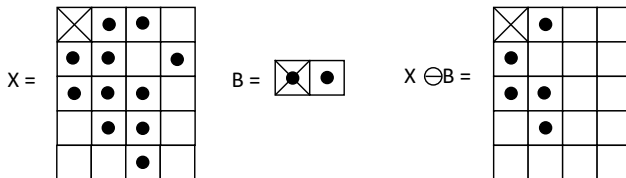
$$X \ominus B = \{d \in E^2; d + b \in X \quad \forall b \in B\} \quad (9)$$

► Example

$$X = \{(0, 1), (0, 2), (1, 0), (1, 1), (1, 3), (2, 0), (2, 1), (2, 2), (3, 1), (3, 2), (4, 2)\} \quad (10)$$

$$B = \{(0, 0), (0, 1)\} \quad (11)$$

$$X \ominus B = \{(0, 1), (1, 0), (2, 0), (2, 1), (3, 1)\} \quad (12)$$



Mathematical morphology

Erosion \ominus properties

- ▶ The most commonly used structuring element - 3x3, containing all 9 points of the eight-neighborhood
 - ▶ objects (lines) of thickness 2 and lonely points disappear
 - ▶ objects are reduced by 1 layer
- ▶ if we subtract its erosion from the original image, we get the outlines of the object
- ▶ Properties
 - ▶ if the origin is contained in a structuring element, it is antiextensive
 $(0,0) \in B \Rightarrow X \ominus B \subseteq X$
 - ▶ invariant with respect to displacement $X_h \ominus B = (X \ominus B)_h$ and $X \ominus B_h = (X \ominus B)_{-h}$
 - ▶ erosion can be expressed as the intersection of shifted point sets $X \ominus B = \bigcap_{b \in B} X_{-b}$

Mathematical morphology

Dual properties of dilation and erosion

- ▶ symmetric set $B^\sim = \{b; -b \in B\}$
- ▶ erosion, unlike dilation, is not commutative
- ▶ dilation and intersection $(X \cap Y) \oplus B \subseteq (X \oplus B) \cap (Y \oplus B)$ and $B \oplus (X \cap Y) \subseteq (B \oplus X) \cap (B \oplus Y)$
- ▶ erosion and intersection $(X \cap Y) \ominus B \subseteq (X \ominus B) \cap (Y \ominus B)$ and $B \ominus (X \cap Y) \supseteq (B \ominus X) \cap (B \ominus Y)$
- ▶ dilation and union $B \oplus (X \cup Y) = (X \cup Y) \oplus B = (X \oplus B) \cup (Y \oplus B)$
- ▶ erosion and union $(X \cup Y) \ominus B \supseteq (X \ominus B) \cup (Y \ominus B)$ and $B \ominus (X \cup Y) \supseteq (X \ominus B) \cap (Y \ominus B)$
- ▶ if we use two structuring elements for dilation / erosion in succession, it does not matter which one we use the first

Mathematical morphology

Opening and Closing

- ▶ combination of dilatation and erosion
- ▶ the resulting image contains less detail
- ▶ **Opening** - erosion followed by dilation

$$X \circ B = (X \ominus B) \oplus B \quad (13)$$

- ▶ **Closing** - erosion followed by dilation

$$X \bullet B = (X \oplus B) \ominus B \quad (14)$$

- ▶ If the image does not change after opening/closing by structuring element B, we say that it is open/closed with respect to B.

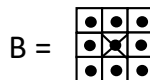
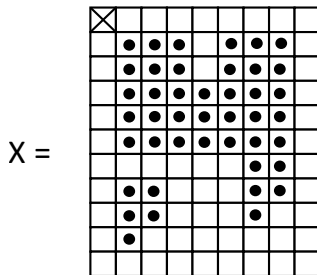
Mathematical morphology

Opening and Closing properties

- ▶ Opening separates objects connected by a narrow neck, removes small details.
- ▶ Closing connects objects that are close to each other, filling small holes and narrow bays
- ▶ The meaning of the terms "small", "narrow", "close" depends on the size of the structuring element.
- ▶ **Closing** - erosion followed by dilation
- ▶ both opening and closing are invariant with respect to the translation
- ▶ opening is an antiextensive projection $X \circ B \subseteq X$
- ▶ closing is an extensive projection $X \subseteq X \circ B$
- ▶ both opening and closing are idempotent, i.e., repeated use of these operations does not change the result

Mathematical morphology

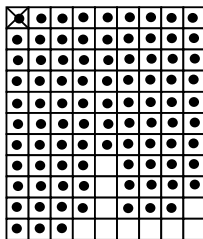
Opening and Closing Example – input image and structuring element



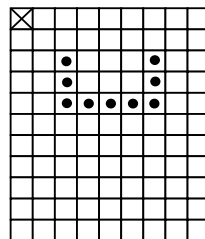
Mathematical morphology

Opening and Closing Example – results of dilation and erosion

$$X \oplus B =$$



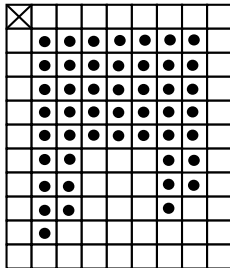
$$X \ominus B =$$



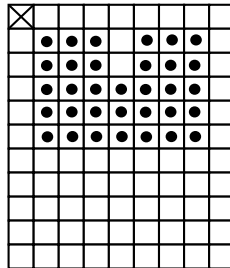
Mathematical morphology

Opening and Closing Example – results of opening and closing

$$X \bullet B = (X \oplus B) \ominus B$$



$$X \circ B = (X \ominus B) \oplus B$$



Mathematical morphology

Opening and Closing Example – results of opening followed by closing (vice versa)

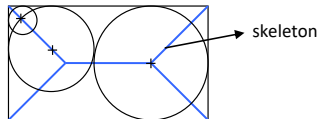
$$(X \bullet B) \circ B = \text{Grid} = (X \circ B) \bullet B$$

In this case, they are equal. (Generally not)

Mathematical morphology

Skeleton

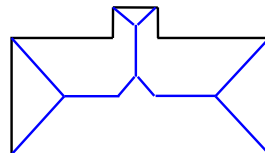
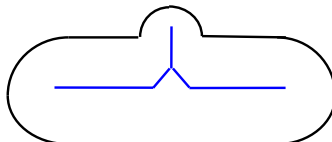
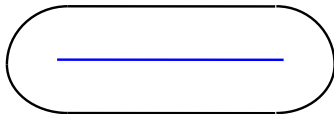
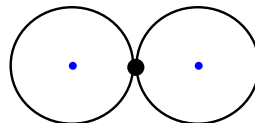
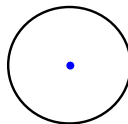
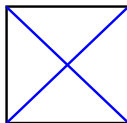
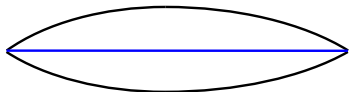
- The skeleton $S(Y)$ is a set of points - the centers of circles, which are contained in Y and touch the boundary Y at least in 2 points.



- The skeleton can be formed by erosions and dilatations, but the skeleton thus obtained can be composed of lines thicker than one point.
- Often the skeleton is replaced by a set processed by sequential homotopic processing

Mathematical morphology

Skeleton - examples



Mathematical morphology

Hit or miss transformation

- ▶ composite structuring element $B = (B_1, B_2)$
- ▶ we are looking for whether $B_1 \subset X$ and at the same time $B_2 \subset X^C$
- ▶ definition

$$X \otimes B = \{x : B_1 \subset X \wedge B_2 \subset X^C\} \quad (15)$$

- ▶ definition using dilation and erosion

$$X \otimes B = (X \ominus B_1) \cap (X^C \ominus B_2) = (X \ominus B_1) \mid (X \oplus B_2^{\sim}) \quad (16)$$

- ▶ where \mid one-sided difference of sets $X \mid Y = X \cap Y^C$

Segmentation

INPUT: INTENSITY IMAGE

OUTPUT: IMAGE DIVIDED INTO AREAS RELATED TO REAL WORLD OBJECTS

Complete segmentation:

- ▶ distinguishable areas related to real world objects.
- ▶ necessary to use knowledge about solved problem.
- ▶ special case: constant objects in front of constant background of known brightness
 - good results without any further knowledge of the problem.

Examples: text in scanned document, blood cells, counting screws

Partial segmentation:

- ▶ areas are homogeneous with respect to particular selected properties (brightness, color, texture, ...)
- ▶ areas can overlay each other
- ▶ necessary to use knowledge about solved problem.

Examples: scene with field and forest viewed from window – areas can not be related to real world objects

Segmentation - Task knowledge

It is necessary to use knowledge about the problem to obtain the best results. For example:

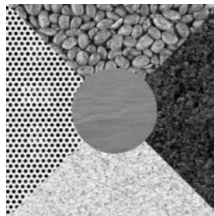
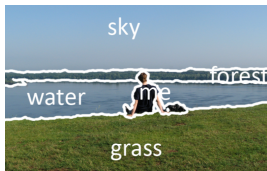
- ▶ defined shape
- ▶ defined position and orientation
- ▶ defined start and end point of the object edge
- ▶ relation between object and other areas

Examples:

- ▶ *searching for ships on the sea (example property: color of background)*
- ▶ *searching for railways and highways in the map (example property: maximum curvature)*
- ▶ *searching for rivers in the map (example property: rivers does not intersect)*

Segmentation - Segmentation approaches

- ▶ brightness approaches – thresholding
- ▶ edges based approaches
- ▶ areas based approaches



Segmentation - Thresholding

- ☺ oldest and simplest approach
- ☺ the most common approach
- ☺ low resource and computing requirements
- ☺ fastest approach – capable to run in real-time
- ☹ threshold determination – not a simple task to perform it automatically
- ☹ can be perform only on particular type of input images (objects and background are easy to distinguish)

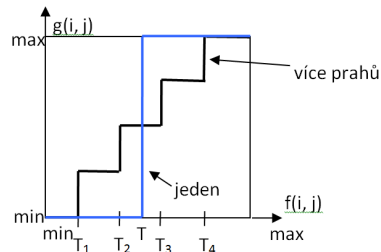
$$g(i,j) = \begin{cases} 1 & \text{for } f(i,j) \geq T \\ 0 & \text{for } f(i,j) < T \end{cases} \quad (17)$$

T – threshold constant

Segmentation - Thresholding

$$g(i,j) = \begin{cases} 1 & \text{pro } f(i,j) \geq T \\ 0 & \text{pro } f(i,j) < T \end{cases} \quad (18)$$

T – threshold constant



Segmentation - Thresholding

Modification: thresholding with a set of known brightness constants

$$g(i, j) = \begin{cases} 0 & \text{for } f(i, j) \in D \\ 1 & \text{otherwise} \end{cases} \quad (19)$$

where D is a set of brightness values representing background.

Examples: blood cells – cytoplasm has particular set of brightness (background is brighter, core is darker)

Modification: thresholding with multiple threshold

$$g(i, j) = \begin{cases} 1 & \text{for } f(i, j) \in D_1 \\ 2 & \text{for } f(i, j) \in D_2 \\ \vdots & \\ n & \text{for } f(i, j) \in D_n \\ 0 & \text{otherwise} \end{cases} \quad (20)$$

where $D_i \cap D_j = \emptyset \ i \neq j$

Segmentation - Thresholding

Modification: partial thresholding

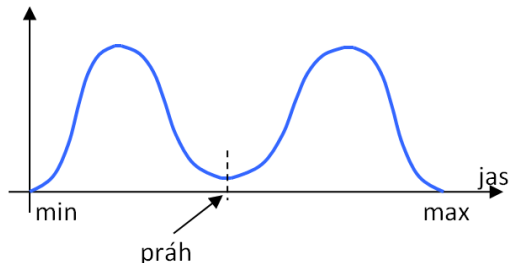
$$g(i, j) = \begin{cases} f(i, j) & \text{for } f(i, j) \in D \\ 0 & \text{otherwise} \end{cases} \quad (21)$$

where D is a set of brightness values related to e.g multiple objects.

- ▶ remove background
- ▶ preserve brightness values in objects
- ▶ $f(i, j)$ **not only** brightness function (e.g. image gradient value, texture property, depth map, color)
- ▶ *demo*

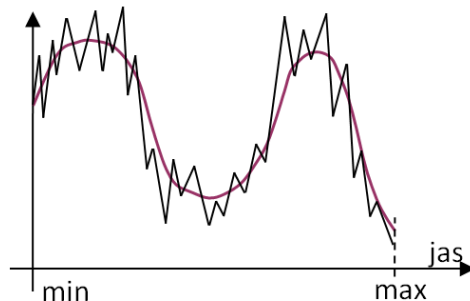
Determination of the threshold

- ▶ basic thresholding is based on known (in advance) threshold
- ▶ input information for threshold determination is usually image histogram
- ▶ how to determine the threshold **automatically**?; "trial and error" approach or;
- ▶ bimodal histogram (2 sufficiently distant maximums)
- ▶ in this case is possible to determine minimum value between maximums.



Histogram smoothing:

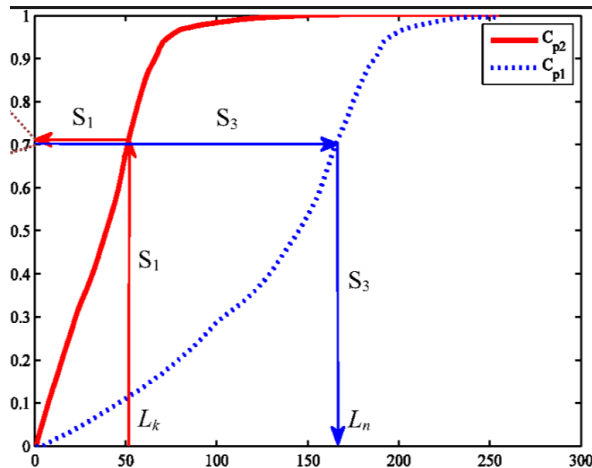
- ▶ we are looking for a local minimum between the two largest sufficiently distant local maxima
- ▶ but often it is not possible to decide unambiguously about the significance of local maxima and minima
- ▶ smoothing suppresses local extremes and ideally provides a bimodal histogram (local averaging - e.g. Gaussian window or median filtering, etc.)



Percentage thresholding

- ▶ we have an a priori knowledge of what percentage of the image area is covered by objects.
- ▶ e.g. the average text coverage of the page area is around 5 %
- ▶ we set the threshold so that just as many percent of the pixels have the color of the objects, the rest the background color.
- ▶ see fig. on the next slide: cumulative histogram for 2x differently lit scene, object covers 70 %

Percentage thresholding



Adaptive thresholding

- ▶ one global threshold value may not be appropriate for certain cases
- ▶ the image may have different lighting conditions in different places
- ▶ in this case the adaptive thresholding calculates the threshold for small regions of the image (sliding window)



Segmentation based on edge detection

Edges places of the image where there is a certain discontinuity, mostly in brightness, but also in color, texture, depth, etc.

Edge image is created by the application of an edge operator.

Border is a description of the edge of a segmented object.

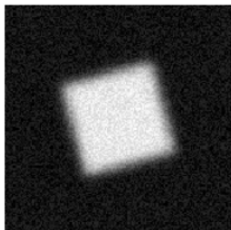
- ▶ The task of segmentation in this case is to join the edges into strings that better match the course of the boundaries
- ▶ A priori information about where the edges are and what their relationships are to other parts of the picture is often used
- ▶ If a priori information is not available, the segmentation method must take into account local properties together with general knowledge specific to the application area.

Edge image thresholding

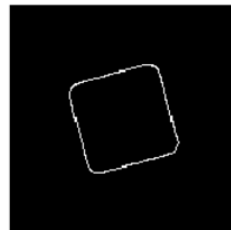
- ▶ usually very few places in the image have zero edge size. The reason is the presence of noise
- ▶ the edge image thresholding method suppresses indistinct edges of small size and preserves only significant edges (the meaning of the words "small", "significant" is related to the size of the threshold)
- ▶ the threshold value can be determined, for example, by percentage thresholding method
- ▶ sometimes post-processing of the result is applied - e.g. omitting **edges** shorter than a certain value
- ▶ see figure on the next slide

Edge image thresholding

noisy image



Canny filter, $\sigma = 1$ Canny filter, $\sigma = 3$



Determining the boundary using knowledge of its position

- ▶ We assume information about the probable **position and shape** of the border, obtained, for example, through higher-level knowledge or as a result of segmentation methods applied to a lower-resolution image.
- ▶ One possibility is to determine the position of the boundary as the position of significant edge cells, which are located near the assumed location of the boundary and which have a direction close to the assumed direction of the boundary at a given location.
- ▶ If a sufficient number of pixels meeting these conditions can be found, a suitable approximation curve is intersected by these points - *refined boundary*
- ▶ see figure on the next slide

Determining the boundary using knowledge of its position

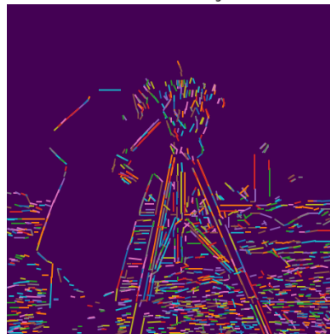
Input image



Canny edges



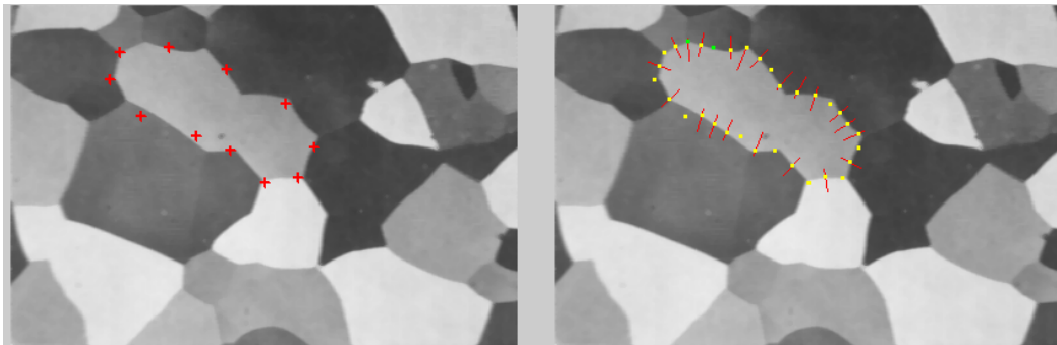
Probabilistic Hough



Gradual division of connectors

- ▶ We will use it if we know the endpoints of the boundary and assume little noise and little curvature of the boundary
- ▶ A possible approach is to gradually divide the connectors of already detected neighboring boundary elements and search for another boundary element on the **normal** guided by the center of this connector.
- ▶ The edge element that is closest to the junction of the points detected so far and has a **above-threshold** edge size is considered a new boundary element, and the iteration process is repeated.
- ▶ see the figure on the next slide

Gradual division of connectors



Region Growing

- ▶ Can be used in noise images where it is difficult to find boundaries
- ▶ An important feature is **HOMOGENITY**
- ▶ Dividing the image into maximum contiguous areas so that these areas are homogeneous in some respects.

Region Growing

Homogeneity criterion

- ▶ based on luminance properties, more complex methods of description or even on the model of the segmented image.
- ▶ we usually require the following conditions to be met for the areas:

1. $H(R_i) = TRUE \quad \text{for } i = 1, 2, \dots, I$
2. $H(R_i \cup R_j) = FALSE \quad \text{for } i, j = 1, 2, \dots, I \quad i \neq j \quad R_i \text{ neighbor } R_j$

Where: I number of areas R_i individual areas $H(R_i)$ the two-valued expression of the homogeneity criterion \rightarrow areas must be (1) homogeneous and (2) maximal

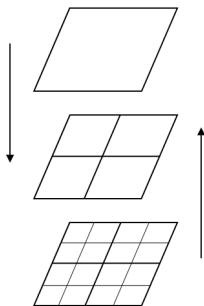
Region Growing - Algorithm

The most natural method of joining regions is based on an initial layout, where each pixel represents a **separate region**, which does not satisfy (2). Furthermore, we always connect two adjacent areas, if the area created by joining these two areas will meet the criterion of homogeneity.

- ▶ The result of the merging depends on the order in which the areas are presented for merging.
- ▶ The simplest methods are based on the initial segmentation of the image into 2x2, 4x4 or 8x8 areas.
- ▶ The description of homogeneity is mostly based on statistical brightness properties (eg brightness histogram in the area).
- ▶ The description of the area is compared using statistical tests with the description of the neighboring area.
 - ▶ when matched, the two areas merge and a new area is created
 - ▶ when no two areas can be merged, the process ends

Split and Merge

- ▶ This method preserves the good properties of both approaches.
- ▶ It uses a pyramidal representation of the image.
- ▶ The areas are square and correspond to an element of a given level of the pyramidal data structure.



1. We will initially specify initial image layout.
2. If for the area R the k -th level of the pyramidal structure $H(R) = FALSE$ (the area is not homogeneous), we divide R into 4 areas ($k + 1$). levels.
3. If there are adjacent areas R_i and R_j such that $H(R_i \cup R_j) = TRUE$, we combine R_i and R_j into one area.
4. If no area can be joined or divided, the algorithm ends

Thank you for your attention!

Questions?



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